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IV. Put angle $B=\theta$. For every value of B there are two triangles whose average arc is $\frac{1}{2}a^2\sin\theta\cos\theta$. Hence,

$$A_4=\tfrac{1}{2}a^2\int_0^{\sin^{-1}\frac{b}{a}}\sin\theta\cos\theta d\theta\div\int_0^{\sin^{-1}\frac{b}{a}}d\theta=\tfrac{1}{4}b^2\div\sin^{-1}\frac{b}{a}.$$

Corollary. If $b=a$, $A_1=a^2/\pi$, and $A^2=a^2/3$. These are double the values found in the solutions of problem 26, as they evidently should be. The values of A_3 and A_4 do not hold when $b=a$, for the reason that while the sum of the areas remains the same the number of triangles is reduced one-half at the moment that $b=a$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let x=third side, A=area, Δ=average area.

$$\therefore\ A=\tfrac{1}{4}\sqrt{(a+b)^2-x^2}\sqrt{x^2-(a-b)^2}.$$

$$\Delta=\int_{a-b}^{a+b}A\,dx\div\int_{a-b}^{a+b}dx=\frac{1}{2b}\int_{a-b}^{a+b}A\,dx.$$

Let $(a+b)^2-x^2=4aby^2$, $\dfrac{4ab}{(a+b)^2}=e^2$.

$$\therefore\ \Delta=\frac{2a^2b}{a+b}\int_0^1\frac{y^2\sqrt{1-y^2}dy}{\sqrt{1-e^2y^2}}=\tfrac{1}{2}a(a+b)\int_0^1\frac{y^2\sqrt{1-e^2y^2}dy}{\sqrt{1-y^2}}$$

$$-\frac{a(a-b)^2}{2(a+b)}\int_0^1\frac{y^2dy}{\sqrt{1-y^2}\sqrt{1-e^2y^2}}=\frac{a+b}{12b}\{(a^2+b^2)E(e)-(a-b)^2F(e)\}.$$

III. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C., and A. P. REED, Clarence, Missouri.

The area of triangle is $\triangle=\frac{1}{2}ab\sin\theta$.

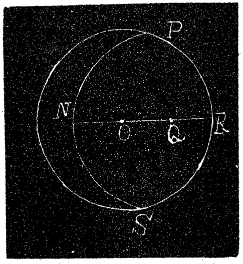
$$\text{Hence, average area}=\tfrac{1}{2}ab\int_0^\pi\sin\theta d\theta\div\int_0^\pi d\theta=\tfrac{1}{2}ab\Big[-\cos\theta\Big]_0^\pi\div\pi=\frac{ab}{\pi}.$$

38. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

Two arrows are sticking in a circular target: show that the chance that their distance is greater than the radius of the target is $3\sqrt{3}/4\pi$. [From *Todhunter's Integral Calculus*, page 335.]

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

Let Q be the position of one arrow. Call the radius of target R, and let $OQ=\rho$.



The area $PNSR=2R^2\cos^{-1}\frac{\rho}{2R}-\rho\sqrt{R^2-(\frac{1}{2}\rho)^2}$.

Then the chance that the second arrow is within the above region is

$$\frac{2}{\pi}\cos^{-1}\frac{\rho}{2R}-\frac{\rho}{\pi R^2}\sqrt{R^2-(\tfrac{1}{2}\rho)^2}.$$

The chance that the first arrow is at a distance ρ from the center is

$$\frac{2\pi\rho d\rho}{\pi R^2}=\frac{2\rho d\rho}{R^2}.$$

The chance that the *two* arrows are as indicated above is

$$\frac{4}{\pi R^2}\cos^{-1}\frac{\rho}{2R}.\rho d\rho-\frac{2}{\pi R^4}\sqrt{R^2-(\tfrac{1}{2}\rho)^2}\,\rho^2 d\rho.$$

The sum of all such chances is

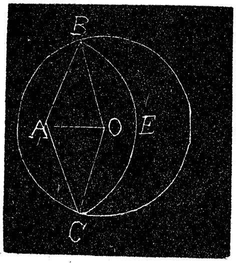
$$\frac{4}{\pi R^2}\int_0^R\rho\cos^{-1}\frac{\rho}{2R}d\rho-\frac{2}{\pi R^4}\int_0^R\sqrt{R^2-(\tfrac{1}{2}\rho)^2}\,\rho^2 d\rho=1-\frac{3\sqrt{3}}{4\pi}.$$

$\therefore$ Chance that the second arrow is *without* the region $PNSR$ is

$$1-\left(1-\frac{3\sqrt{3}}{4\pi}\right)=\frac{3\sqrt{3}}{4\pi}.$$

II. Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

Let O be the center of the target and A the position of one of the arrows. Then if the distance between the arrows is greater than the radius, a, the other arrow must lie outside the arc BEC drawn from A as center and radius a. If $AO=x$, the area of the surface outside of the arc BEC is



$$S=2a^2\sin^{-1}\left(\frac{x}{2a}\right)+\tfrac{1}{2}x(4a^2-x^2)^{\frac{1}{2}}.$$

The probability that the one arrow is at the distance x from the center is $2\pi x dx\div a^2\pi=2xdx\div a^2$. The probability that the other is on the surface outside the arc BEC is $S\div a^2\pi$. Hence the required probability is

$$P=\frac{2}{a^4\pi}\int_0^a Sxdx. \quad \text{Put}\, x=2a\sin\theta.$$

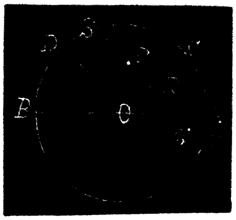
$$\text{Then } P=\frac{16}{\pi}\int_0^{\frac{1}{6}\pi}(\theta+\sin\theta\cos\theta)\sin\theta\cos\theta=3\sqrt{3}/4\pi.$$

III. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let P, Q be the arrows, $SQ=x$, $PQ=y$, $ST=u$, $OR=z$, $\angle DOB=\theta$, $OA=a$.

An element of area at Q is $dzdx$; at P, $yd\theta dy$.

The limits of x and 0 are $u-a$; of y, $u-x$ and a, and doubled ; of z, 0 and $\frac{1}{2}a\sqrt{3}$, and doubled ; of θ, 0 and $\frac{1}{2}\pi$, and doubled. $\varDelta$=chance, $u=2\sqrt{a^2-z^2}$.



$$\therefore\ \varDelta=\frac{8}{\pi^2a^4}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}a\sqrt{3}}\int_0^{u-a}\int_a^{u-x}d\theta dzydxdy$$

$$=\frac{4}{\pi^2a^4}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}a\sqrt{3}}\int_0^{u-a}\{(u-x)^2-a^2\}d\theta dzdx$$

$$=\frac{4}{3\pi^2a^4}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}a\sqrt{3}}(u^3+2a^3-3a^2u)d\theta dz=\frac{3\sqrt{3}}{2\pi^2}\int_0^{\frac{1}{2}\pi}d\theta=\frac{3\sqrt{3}}{4\pi}.$$

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MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

34. Proposed by THOS. U. TAYLOR, C. E., M. C.; Department of Engineering, University of Texas, Austin, Texas.

Given a variable parallelogram $ABCP$, where P remains fixed. A moves on an irregular plane curve (closed) and C moves on another irregular plane curve (closed) whose plane is parallel to the plane of (A) curve. The generator PC moves completely around and returns to its initial position, AB always moving parallel to PC, and, of course, returns to its initial position. If distance between planes (A) and (C)$=h$, show by elementary mathematics and without using theorem of Koppe that volume of solid generated by variable parallelogram $ABCP=\frac{1}{2}h$ (area generated by AP+area generated by BC).